3. L. M. Levin, Investigations in the Physics of Coarsely Dispersed Aerosols [in Russian], Izd. Akad. Nauk SSSR, Moscow (1961).
4. V. M. Voloshchuk, Introduction to the Hydrodynamics of Coarsely Dispersed Aerosols [in Russian], Gidrometeoizdat, Leningrad (1971).
5. S. A. Morsi and A. J. Alexander, "An investigation of particle trajectories in two-phase flow systems," J. Fluid Mech., 55, Pt. 2 (1972).
6. A. M. Mkhitaryan, Yu. V. Sukharnikov, and Yu. G. Koval', "Study of the settling of drops on cylindrical bodies moving in an aerosol cloud," in: Aerodynamics [in Russian], No. 3, Kiev (1977).
7. Yu. M. Tsirkunov, "Effect of a viscous boundary layer on the settling of particles in the flow of a gas suspension past a sphere," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 1 (1982).
8. Yu. M. Tsirkunov, "Approximation of the parameter fields of a viscous carrying medium in the study of the flow of a dispersed mixture with large Reynolds numbers past a blunt body," in: Dynamics of Inhomogeneous and Compressible Media [in Russian], Izd. Leningr. Gos. Univ., Leningrad (1984).
9. J. Happel and H. Brenner, Hydrodynamics at Low Reynolds Numbers [Russian translation], Mir, Moscow (1976).
10. R. I. Nigmatulin, Foundations of the Mechanics of Heterogeneous Media [in Russian], Nauka, Moscow (1978).
11. M. Van Dyke, "High approximations in boundary-layer theory. Part 1. General analysis," J. Fluid Mech., 14, Pt. 2 (1962).
12. N. E. Kochin, I. A. Kibel', and N. V. Roze, Theoretical Hydromechanics [in Russian], Fizmatgiz, Moscow (1963), Pt. 1.
13. G. Shlikhting, Boundary Layer Theory [in Russian], Nauka, Moscow (1974).
14. R. V. Hemming, Numerical Methods [Russian translation], Nauka, Moscow (1972).
15. S. H. Lee, R. S. Chadwick, and L. G. Leal, "Motion of a sphere in the presence of a plane interface. Part 1. An approximate solution by generalization of the method of Lorentz," J. Fluid Mech., 93, Pt. 4 (1979).
16. A. J. Goldman, R. G. Cox, and H. Brenner, "Slow viscous motion of a sphere parallel to a plane wall. II. Couette flow," Chem. Eng. Sci., 22, No. 4 (1967).
17. P. A. Kouzov, Foundations of the Analysis of the Dispersed Composition of Industrial Dusts and Pulverized Materials [in Russian], Khimiya, Leningrad (1971).

SUPERSONIC FLOW OF A GAS SUSPENSION NEAR A WEDGE IN THE PRESENCE OF REFLECTED PARTICLES

V. D. Sarychev, A. P. Trunev, and V. M. Fomin

UDC 532.529.5

Supersonic flow, perturbed by the interaction of the gas with a cloud of monodispersed particles, near a wedge is studied. The exact solution of the problem of the motion of particles behind an oblique shock and particles specularly reflected from the surface of the wedge is given. These results are used to determine the perturbations of the gasdynamic parameters and forces acting on the wedge in a two-phase flow. The effects of the particles on the flow in two different situations are compared: In one situation the particles stick to the surface of the wedge; in the other situations they reflect elastically, and form a layer of dust with a sharp contact boundary.

The problem of the perturbed gas flow behind an oblique shock was previously studied in gasdynamics [1] and in the dynamics of a radiating gas [2]. The problem of supersonic two-phase flow near a wedge was studied on the basis of the linear theory in [3] and by numerical methods in [4]. The solution of the problem of the motion of a particle behind an oblique shock [5] and of a reflected particle [6] are known. The exact solution of the problem of the motion of a cloud of particles behind an oblique shock was found in [7].

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 102-110, September-October, 1985. Original article submitted March 19, 1984.

 0021-8944/85/2606-0694$09.50

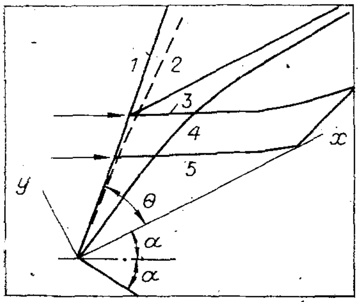

Fig. 1

1. Motion of a Dust Cloud in the Presence of Supersonic Flow near a Wedge. We shall study the aerodynamic situation arising when a supersonic flow of a gas suspension impinges against a wedge whose half-angle is equal to α; the flow can impinge against a flat plate, in which case α is the angle of attack. The general pattern of the flow in the case of a small admixture of monodispersed particles is shown in Fig. 1, where 1 is the position of the oblique (unperturbed) shock, 2 is the streamline of the gas in the main flow, 3 is the trajectory of the particle which does not reach the surface of the wedge, 4 is the limiting trajectory below which the cloud of reflected particles moves, 5 is the trajectory of a particle undergoing a collision with the wedge, and x, y are the axes of a Cartesian coordinate system; the broken line shows the position of the perturbed shock.

Under the assumption that the relative flow rate of the condensed phase in the incident flow is small ($\rho_{p\infty}u_{p\infty}/\rho_\infty u_\infty \ll 1$), we shall study the problem of finding the parameters of the supersonic flow of the gas mixture in the entire flow region. We shall solve this problem by iterations, similar to the trajectory method for calculating two-phase flows [8]. At the first step we assume that the gas flow is free of particles, when the solution of the gasdynamic equations reduces to conditions on the oblique shock. The motion of the particles in this approximation occurs against the background of the constant flow of gas and reduces to relaxation from one equilibrium state to another. At the next step the interaction of the gas with the particles is "switched on," and the intensity of the interaction is determined from the conservation laws for mass, momentum, and energy for the mixture as a whole. At the second step the motion of the particles is assumed to be known from the solution obtained at the first step. Because $\delta_0 \ll 1$, the solution of the gasdynamic equations can be reduced to finding the acoustic perturbations caused by the volume sources. By means of the above-described procedure the gasdynamic parameters and the particle flux density are determined to within $\sim\delta_0^2$, and the temperature and velocity of the condensed phase are determined to within $\sim\delta_0$. It is, however, important that the forces acting on the wedge in the flow of the gas suspension be determined to within $\sim\delta_0^2$, since the flow rate of the discrete phase is of the order of $\sim\delta_0$. In this model it is necessary to take into account the effect of the reflected particles on the state of the gas flow, but the collision of the incident and reflected particles may be neglected, because the intensity of their interaction is of the order of $\sim\delta_0^2$.

Thus according to the foregoing plan we shall study the motion of a cloud of particles behind an oblique shock. To describe the motion of a large number of particles we shall use the continuum representations. The corresponding equations have the form [9]

$$\begin{gathered}\partial/\partial x(\rho_p u_p) + \partial/\partial y(\rho_p v_p) = 0,\\ (u_p\partial/\partial x + v_p\partial/\partial y)u_p = (3\rho_0 C_D/4\rho_s d_p)W(u_0 - u_p),\\ (u_p\partial/\partial x + v_p\partial/\partial y)v_p = -(3\rho_0 C_D/4\rho_s d_p)Wv_p,\\ (u_p\partial/\partial x + v_p\partial/\partial y)T_p = (6c_p\mu_0\,\mathrm{Nu}/c_s\rho_s d_p^2\,\mathrm{Pr})(T_0 - T_p),\end{gathered} \tag{1.1}$$

where $W = \sqrt{(u_0 - u_p)^2 + v_p^2}$; ρ_0, u_0, T_0, μ_0 are the density, velocity, temperature, and viscosity of the gas behind the oblique shock; c_p is the specific heat capacity of the gas; ρ_s, c_s are the density and specific heat capacity of the particle material; d_p is the diameter of the particle; Nu is the Nusselt number; and Pr is the Prandtl number.

We shall further assume that the drag C_D depends only on the Mach and Reynolds numbers of the relative motion $C_D = C_D(\overline{M}, \overline{Re})$, where $\overline{M} = W/(\gamma RT_0)$ and $\overline{Re} = Wd_p\rho_0/\mu_0$, while the heat-transfer law has the form $\mathrm{Nu} = \mathrm{Nu}(\overline{M}, \overline{Re}, \mathrm{Pr})$.

We shall give the boundary conditions for the system (1.1) on the line of the shock, assuming that the velocity and temperature of the dust particles are the same as that of the gas in the incident flow, i.e., we are talking about the motion of some body through a cloud of dust in the atmosphere. Thus along the line $y = x \tan\theta$ the following conditions hold:

$$\rho_p = \rho_{p\infty},\ u_p = q_\infty \cos\alpha,\ v_p = -q_\infty \sin\alpha,\ T_p = T_\infty, \tag{1.2}$$

where θ is the angle of inclination of the shock wave relative to the surface of the wedge; q_∞, T_∞ are the modulus of the velocity and temperature of the gas in the incident flow.

We note that the first and fourth equations of the system (1.1) are linear in l_p and T_p, respectively. It is therefore necessary to solve the nonlinear system of the two remaining equations; this was done in [7]. We recall the basic idea of the solution of the problem. The system under study and the corresponding boundary conditions do not contain arbitrary functions and contain one parameter with the dimensions of length l_p. Since the coordinates x and y are equivalent in this problem, the solution cannot depend on x and y separately; it depends only on some linear combination of these variables. The variable $\xi = x\tan\theta - y$ is such a combination. It follows from a dimensional analysis that the solution of the problem depends on the dimensionless ratio ξ/l_p. This holds in reality, but only in integral form. The solution of the problem (1.1) and (1.2) can be represented in the form (cf. [7])

$$\begin{gathered}\rho_p = \rho_{p\infty}/[\varepsilon + (1-\varepsilon)\eta],\ u_p = u_0 + (q_\infty\cos\alpha - u_0)\eta,\\ v_p = -(q_\infty \sin\alpha)\eta,\ T_p = T_0 - (T_0 - T_\infty)e^{-J(\eta)},\end{gathered} \tag{1.3}$$

where $\varepsilon = \rho_\infty/\rho_0$ is the ratio of the gas densities before and after the shock. The functions $\eta(\xi)$ and $J(\eta)$ satisfy the equations

$$\begin{gathered}\xi\sin\alpha/\sin(\theta+\alpha) = \int_\eta^1 [\varepsilon + (1-\varepsilon)s]\, l_p(s)\, ds/s^2,\\ J(\eta) = (8c_p/c_s \mathrm{Re}_0 \mathrm{Pr})\int_\eta^1 \mathrm{Nu}(s\,\mathrm{Re}_0, s\mathrm{M}_0, \mathrm{Pr})\, C_D^{-1}(s\,\mathrm{Re}_0, s\,\mathrm{M}_0)\, ds/s^2,\\ \mathrm{Re}_0 = q_\infty d_p \rho_0 \sin\alpha/(\mu_0\cos\theta),\ \mathrm{M}_0 = \mathrm{M}_\infty T_\infty \sin\alpha/(T_0\cos\theta),\\ l_p(s) = 4\rho_s d_p/(3\rho_0 C_D(s\,\mathrm{Re}_0, s\mathrm{M}_0)).\end{gathered} \tag{1.4}$$

We shall clarify the origin of the complicated arguments in the integrands in (1.4). According to (1.3), the modulus of the velocity is given by $W = ((u_p - u_0)^2 + v_p^2)^{1/2} = \eta q_\infty \sin\alpha/\cos\theta$. Therefore $\overline{M} = \eta M_0$, $\overline{Re} = \eta Re_0$ and correspondingly

$$C_D = C_D(\eta\,\mathrm{Re}_0, \eta\mathrm{M}_0),\ \mathrm{Nu} = \mathrm{Nu}(\eta\,\mathrm{Re}_0, \eta\mathrm{M}_0, \mathrm{Pr}).$$

On the surface of the wedge the particles have a velocity component normal to the surface $v_p = -(q_\infty \sin\alpha)\eta(x\tan\theta)$. These particles collide with the surface and either stick to it, for example, if they are drops of liquid, or are reflected from it and continue their motion, if they are solid particles. Let us study the problem of the motion of a cloud of particles reflected specularly from the surface. We shall distinguish the parameters of the flow of reflected particles by a prime; then the boundary conditions on the surface of the wedge for them have the form (condition of specular reflection)

$$\rho_p' = \rho_p,\quad u_p' = u_p,\quad v_p' = -v_p,\quad T_p' = T_p. \tag{1.5}$$

Let us assume that the motion of reflected particles is described by the system of equations (1.1), in which all parameters are primed, after which we denote the system by (1.1'). We note that the system of equations (1.1) is invariant relative to the simultaneous substitution $y \to -y$, $v_p \to -v_p$. From here and from the boundary conditions (1.5) it follows that the solution of the problem (1.1) and (1.5) can be represented in the form (1.3), but with ξ replaced by the "mirror" variable $\xi' = x\tan\theta + y$ and simultaneously $v_p \to -v_p'$:

$$\begin{gathered}\rho_p' = \rho_{p\infty}/[\varepsilon + (1-\varepsilon)\eta'],\quad u_p' = u_0 + (q_\infty\cos\alpha - u_0)\eta',\\ v_p' = (q_\infty\sin\alpha)\eta',\quad T_p' = T_0 - (T_0 - T_\infty)e^{-J'},\end{gathered} \tag{1.6}$$

where $J' = J(\eta')$, while the function $\eta'(\xi')$ satisfies the equation

$$\xi'\sin\alpha/\sin(\alpha+\theta) = \int_{\eta'}^1 [\varepsilon + (1-\varepsilon)s]\, l_p(s)\, ds/s^2. \tag{1.7}$$

All arguments preceding the derivation of the solution of the problem (1.1) and (1.2) can be successfully replaced by a group analysis of the corresponding equations (see [10]). This remark also concerns the method used to find the solution of the problem (1.1) and (1.5). The existence of a discrete symmetry group (mirror symmetry) facilitates finding the solution of the last problem, but this property of the solutions (1.6) is destroyed by any perturbation of the boundary conditions (1.5). From the physical viewpoint, the conditions $u_p' = \lambda_1 u_p$, $v_p' = -\lambda_2 v_p$, describing the partial loss of momentum in a collision (λ_1, λ_2 are the coefficients of restitution of the velocity in the collision), are more acceptable. The appearance of arbitrary coefficients in the condition (1.5) destroys the symmetry of the problem, whose solution in this case cannot be reduced to quadratures.

It is necessary to establish the form of the trajectories of both the incident and reflected particles. If the coordinates of the trajectory of an incident particle are denoted by x_s, y_s, then from (1.4) and the definition of the trajectory we have

$$dx_s \operatorname{tg}\theta - dy_s - [\varepsilon + (1-\varepsilon)s]l_p(s)\sin(\theta+\alpha)ds/s^2\sin\alpha = 0,$$
$$u_p dy_s - v_p dx_s = 0.$$

From here we find an expression for the trajectory in a parametric form:

$$x_s = x_0 - \int_{s_0}^{s} u_p(s)\, l_p(s)\cos\theta\, ds/(s^2 q_\infty \sin\alpha),$$

$$y_s = y_0 - \int_{s_0}^{s} v_p(s)\, l_p(s)\cos\theta\, ds/(s^2 q_\infty \sin\alpha),$$

where $u_p(s) = u_0 + (q_\infty \cos\alpha - u_0)s$; $v_p(s) = -(q_\infty \sin\alpha)s$. Analogously, for trajectories of reflected particles we obtain

$$x_s' = x_1 + \int_{s}^{s_1} u_p'(s)\, l_p(s)\cos\theta\, ds/(s^2 q_\infty \sin\alpha),$$

$$y_s' = \int_{s}^{s_1} v_p'(s)\, l_p(s)\cos\theta\, ds/(s^2 q_\infty \sin\alpha), \qquad (1.8)$$

$$u_p'(s) = u_p(s), \quad v_p'(s) = -v_p(s).$$

Since the motion of the reflected particle depends on its motion before the collision with the surface, we require that $x_s(s_1) = x_1$. On the other hand, from the fact that the particle passed through the shock in the course of its motion, it follows that $y_0 = x_0 \tan\theta$, $s_0 = 1$.

Setting in (1.8) $x_1 = 0$, $s_1 = 1$, we find the equation of the limiting trajectory $y_s^*(x)$, below which the cloud of reflected particles moves (curve 4 in Fig. 1). The slope of the limiting trajectory near the tip of the wedge is given by $dy_s^*/dx(0) = \tan\alpha$ (in the case of specular reflection the angle of incidence is equal to the angle of reflection). If, therefore, the inclination of the shock wave relative to the surface of the wedge is less than the inclination of the limiting trajectory ($\theta < \alpha$), then some relative fraction of the particles is reflected straight into the incident flow. In this case, the solution obtained above for the problem of the motion of reflected particles is not valid. We shall ignore this possibility below, and we shall set simply $\theta \geqslant \alpha$.

2. Interaction of Acoustic Disturbances with a Cloud of Particles. In order to take into account systematically the effect of a large number of particles on the state of the gas flow, we shall use the equations of motion of a non-single-phase mixture, written in the form of conservation laws:

$$\partial/\partial x(\rho u + \rho_p u_p + \rho_p' u_p') + \partial/\partial y(\rho v + \rho_p v_p + \rho_p' v_p') = 0,$$

$$\partial/\partial x(\rho u^2 + p + \rho_p u_p^2 + \rho_p' u_p'^2) + \partial/\partial y(\rho uv + \rho_p u_p v_p + \rho_p' u_p' v_p') = 0,$$

$$\partial/\partial x(\rho uv + \rho_p u_p v_p + \rho_p' u_p' v_p') + \partial/\partial y(\rho v^2 + p + \rho_p v_p^2 + \rho_p' v_p'^2) = 0,$$

$$\partial/\partial x(\rho ue + \rho_p u_p e_p + \rho_p' u_p' e_p') + \partial/\partial y(\rho ve + \rho_p v_p e_p + \rho_p' v_p' e_p') = 0,$$

$$e = \gamma p/\rho(\gamma-1) + (u^2+v^2)/2, \quad e_p = c_s T_p + (u_p^2 + v_p^2)/2,$$

$$e_p' = c_s T_p' + (u_p'^2 + v_p'^2)/2 \text{ (}p\text{ is the gas pressure).} \quad (2.1)$$

Evaluating the relative contribution of the terms with the index p to the dynamics of the carrying gas, we find that this contribution is proportional to the relative flow rate of the particles, i.e., $\sim\delta_0$. Therefore, when $\delta_0 \ll 1$, the problem can be viewed as the perturbed motion of gas in an acoustic formulation. We linearized the gasdynamic parameters using the formulas

$$\rho = \rho_0(1 + \delta_0\tilde{\rho}),\ u = u_0(1 + \delta_0\tilde{u}),\ v = u_0\delta_0\tilde{v},$$
$$p = p_0 + \rho_0 u_0^2\delta_0\tilde{p},\ e = e_0 + u_0^2\delta_0\tilde{e}.$$

We substitute these expressions into (2.1) and express the terms with the index p in accordance with the model adopted for the calculation in the form (1.3) and (1.6). Then, after some simplifications and dropping terms which are second and higher order infinitesimals in δ_0, we obtain the well-known system of equations of supersonic acoustics [1, 2]:

$$\begin{gathered}
\partial\tilde{u}/\partial x + \partial\tilde{\rho}/\partial x + \partial\tilde{v}/\partial y = 0,\\
\partial\tilde{u}/\partial x + \partial\tilde{p}/\partial x = (\mathrm{tg}^2\,\theta/u_0)\left(dv_p/d\xi + \sigma dv_p'/d\xi'\right),\\
\partial\tilde{v}/\partial x + \partial\tilde{p}/\partial y = -(\mathrm{tg}\,\theta/u_0)\left(dv_p/d\xi + \sigma dv_p'/d\xi'\right),\\
\partial\tilde{p}/\partial x - (1/\mathrm{M}^2\gamma)\,\partial\tilde{\rho}/\partial x + (\gamma - 1)/\gamma\partial\tilde{u}/\partial x = -\frac{(\gamma-1)\,\mathrm{tg}^2\theta}{\gamma u_0^2}\left(\frac{de_p}{d\xi} + \sigma\frac{de_p'}{d\xi'}\right).
\end{gathered} \quad (2.2)$$

Here M is Mach's number behind the oblique shock; σ is the unit step function:

$$\sigma = \begin{cases} 1, & y \leqslant y_s^*(x),\\ 0, & y > y_s^*(x); \end{cases}$$

$y_s^*(x)$ is the limiting trajectory. The system of equations (2.2) must be supplemented by boundary conditions on the shock, on the surface of the wedge, and also on the line $y = y_s^* \times (x)$, which is a surface of weak discontinuity of the gasdynamic parameters. The conditions on the shock are obtained by linearization of the Rankine–Hugoniot conditions [1, 2]:

$$\tilde{\rho} = \gamma\mathrm{M}^2\tilde{p} + (\gamma - 1)\mathrm{M}^2\tilde{u},\ \tilde{p} = n\tilde{v},$$
$$\tilde{u} = -[1 + (1/\varepsilon - 1)m]\tilde{v}\,\mathrm{tg}\,\theta,\ \tilde{\theta} = m\tilde{v},$$

where

$$n = \frac{[2\varepsilon - (1-\varepsilon)(\gamma-1)\,\mathrm{M}^2\sin^2\theta]\,\mathrm{tg}\,\theta}{\varepsilon + (\mathrm{M}^2 - 1)\,\mathrm{tg}^2\,\theta - (1-\varepsilon)\,\gamma\mathrm{M}^2\sin^2\theta};$$
$$m = \frac{n - 1 + [\gamma n - \gamma + 1]\,\mathrm{M}^2\sin^2\theta}{(1/\varepsilon - 1)\,[2 + (\gamma - 1)\,\mathrm{M}^2]\sin^2\theta\,\mathrm{tg}\,\theta};$$

$\tilde{\theta}$ is the perturbation of the angle of inclination of the shock wave. The two other conditions have the form

$$\tilde{v} = 0 \text{ at } y = 0,$$
$$\tilde{v}(x, y_s^* + 0) = \tilde{v}(x, y_s^* - 0),\quad \tilde{p}(x, y_s^* + 0) = \tilde{p}(x, y_s^* - 0),$$
$$\tilde{\rho}(x, y_s^* + 0) = \tilde{\rho}(x, y_s^* - 0),\quad \tilde{u}(x, y_s^* + 0) = \tilde{u}(x, y_s^* - 0)$$
$$\text{at } y = y_s^*(x).$$

The last conditions express the property of continuity of the gasdynamic parameters at the surface of the weak discontinuity. The entire region of the flow behind the oblique shock is divided naturally into two regions: D_1 (σ = 0), D_2 (σ = 1). In the region D_1 the solution of Eqs. (2.2) for perturbations $\tilde{p}$ and $\tilde{v}$ can be represented in the form

$$\tilde{p} = \Phi_1(\Psi_+) + \Phi_2(\Psi_-) + \hat{p}(\xi),\ \tilde{v} = \omega(\Phi_2(\Psi_-) - \Phi_1(\Psi_+)) + \hat{v}(\xi),$$

where $\omega = \sqrt{\mathrm{M}^2 - 1}$; $\Psi_\pm = x \pm \omega y$; $\hat{p}(\xi)$, $\hat{v}(\xi)$ are particular solutions of the problem, which are easily found because of the special form of the right sides of Eqs. (2.2). Assuming that all functions sought depend only on ξ, we find

$$\hat{\rho}(\xi) = \hat{v}(\xi)/\sin\theta\cos\theta,$$
$$\hat{u}(\xi) = -\hat{v}(\xi)\,\mathrm{tg}\,\theta,\ \hat{p}(\xi) = \hat{v}(\xi)\,\mathrm{tg}\,\theta + v_p(\xi)\,\mathrm{tg}\,\theta/u_0,$$

$$\hat{v}(\xi) = \frac{(\gamma - 1)(e_p - e_{p0})\,\mathrm{ctg}\,\theta + \gamma u_0 v_p}{[(M \sin\theta)^{-2} - 1]\, u_0^2},$$

where $e_{p0} = c_s T_0 = u_0^2/2$.

Using the conditions on the shock wave, we express Φ_1 and Φ_2 in terms of one function:

$$\tilde{p} = \Phi(\Psi_-) + \lambda_1 \Phi(\lambda_2 \Psi_+) + \hat{p}(\xi) + C,$$
$$\tilde{v} = \omega(\Phi(\Psi_-) - \lambda_1 \Phi(\lambda_2 \Psi_+)) + \hat{v}(\xi) - \omega C, \tag{2.3}$$

where $\lambda_1 = (\omega n - 1)/(\omega n + 1)$; $\lambda_2 = (1 - \omega \tan\theta)/(1 + \omega \tan\theta)$; $C = [n\hat{v}(0) - \hat{p}(0)]/(1 + \omega n)$. In the region D_2 the solutions for the perturbations $\tilde{p}$ and $\tilde{v}$ are represented in the form

$$\tilde{p} = \varphi_1(\Psi_+) + \varphi_2(\Psi_-) + \hat{p}(\xi) + \hat{p}'(\xi'),$$
$$\tilde{v} = \omega(\varphi_2(\Psi_-) - \varphi_1(\Psi_+)) + \hat{v}(\xi) + \hat{v}'(\xi'),$$

where

$$\hat{p}'(\xi') = -\hat{v}' \,\mathrm{tg}\,\theta - v_p' \,\mathrm{tg}\,\theta / u_0;$$
$$\hat{v}'(\xi') = -\frac{(\gamma - 1)(e_p' - e_{p0})\,\mathrm{ctg}\,\theta - \gamma u_0 v_p'}{[(M \sin\theta)^{-2} - 1]\, u_0^2}.$$

From the condition of impermeability at the surface of the wedge, we can express φ_1 and φ_2 in terms of one function:

$$\tilde{p} = \varphi(\Psi_+) + \varphi(\Psi_-) + \hat{p}(\xi) + \hat{p}'(\xi'),$$
$$\tilde{v} = \omega(\varphi(\Psi_-) - \varphi(\Psi_+)) + \hat{v}(\xi) + \hat{v}'(\xi').$$

Joining the functions determined in the regions D_1 and D_2 on the line $y = y_s^*(x)$, we obtain the following functional equations:

$$\varphi(t) = \Phi(t) - [\omega \hat{p}'(\xi_+) + \hat{v}'(\xi_+)]/2\omega,$$
$$\Phi(t) - \lambda_1 \Phi(\lambda_2 t) = C + (\hat{p}'(\xi_-) - \hat{p}'(\xi_+))/2 + (\hat{v}'(\xi_-) + \hat{v}'(\xi_+))/2\omega, \tag{2.4}$$
$$\omega \xi_\pm = \pm t + x_\pm (\omega \,\mathrm{tg}\,\theta \mp 1), \quad x_\pm \pm \omega y_s^*(x_\pm) - t = 0.$$

If the particles stick to the surface of the wedge, then the solution (2.3) is valid in the entire region of the flow. In this case, the determining functional equation has the usual form for perturbation theory [1, 2]:

$$\Phi_0(x) - \lambda_1 \Phi_0(\lambda_2 x) = C - \hat{v}(x \,\mathrm{tg}\,\theta)/\omega, \tag{2.5}$$

where the index 0 distinguishes the case of an inelastic collision.

Thus the solution of the problem of the interaction of acoustic perturbations with a cloud of particles in supersonic flow over a wedge reduces to the solution of the functional equations (2.4) and (2.5) (this is done by numerical methods).

3. For some model laws of drag and heat transfer the integrals in expressions (1.4) and (1.7) can be calculated exactly [7]. In the numerical calculations we assumed, following [9], that the drag and heat-transfer coefficients are equal to

$$C_D(\overline{\mathrm{Re}}, \overline{\mathrm{M}}) = C_D^0 (1 - 0.45\,\mathrm{M} + 4.84\,\mathrm{M}^2 - 9.73\,\mathrm{M}^3 + 6.94\,\mathrm{M}^4)/(1 + 1.2\,\mathrm{M} C_D^0)^{1/2},$$
$$\mathrm{Nu}(\overline{\mathrm{Re}}, \overline{\mathrm{M}}) = \mathrm{Nu}^0/[1 + 3.42\,\overline{\mathrm{M}}\,\mathrm{Nu}^0/\overline{\mathrm{Re}}\,\mathrm{Pr}], \quad \mathrm{Nu}^0 = 2 + 0.459\,\overline{\mathrm{Re}}^{0.55}\,\mathrm{Pr}^{0.33},$$
$$C_D^0 = 21.1/\mathrm{Re} + 6.3/\sqrt{\overline{\mathrm{Re}}} + 0.25.$$

Figure 2 shows the results of calculations illustrating the effect of Re_0 on the dynamic (curves 1-3) and thermal (curves 4-6) relaxation of the particles behind an oblique shock. The function $\eta(\xi)$ (curves 1-3) and the reduced temperature $\overline{T} = (T_p - T_\infty)/(T_0 - T_\infty)$ (curves 4-6) are plotted along the ordinate axis. The calculations were carried out for $c_p/c_s = 0.46$, $\mathrm{Pr} = 0.65$, $M_\infty = 1.7$, $\alpha = 15°$. The curves 1-3 (4-6) correspond to $\mathrm{Re}_0 = 10^0$, 10^2, 10^3. The parameter $l_p(1) = 4\rho_s d_p / 3\rho_0 C_D(\mathrm{Re}_0, M_0)$.

To find the function $\Phi(t)$ [or $\Phi_0(x)$] the solution of the corresponding equation was represented in the form [1]

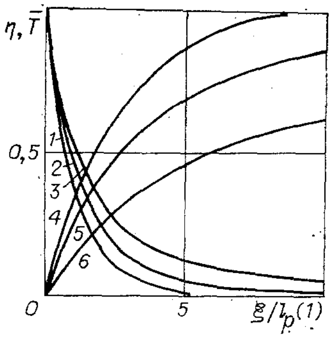

Fig. 2

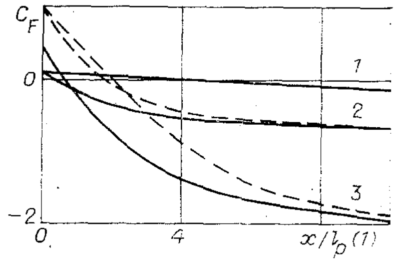

Fig. 3

$$\Phi(t) = \sum_{k=0}^{\infty} \lambda_1^k Q(\lambda_2^k t), \tag{3.1}$$

where Q is the right side of Eq. (2.4) [or (2.5)]. It is well known that the coefficient of reflection of the perturbations from the shock λ_1 is small everywhere except at singular points, in the vicinity of which $M \to 1$. This question is studied in detail in [1]. In order to find the solution with the required accuracy when $\lambda_1 \ll 1$, it is sufficient to retain only several terms in the series (3.1) (it is easy to verify that Q is everywhere a bounded function). To monitor the accuracy of the calculations we used the asymptotic formulas, valid for the case when the particles are reflected in the case that the particles stick:

$$\begin{gathered}
\lim_{y\to\infty} \tilde{\theta} = \lim_{y\to\infty} \tilde{\theta}_0 = m\hat{v}(0), \\
\lim_{x\to\infty} \tilde{p}(x, 0) = \lim_{x\to\infty} \tilde{p}_0(x, 0) = (n - \operatorname{tg}\theta)\hat{v}(0) + \sin\theta \sin\alpha/\cos(\theta+\alpha), \\
\hat{v}(0) = [(\gamma - 1)(e_{p\infty} - e_{p0})\operatorname{ctg}\theta - \gamma u_0 q_\infty \sin\alpha]/((M\sin\theta)^{-2} - 1)\,u_0^2, \\
e_{p\infty} = c_s T_\infty + q_\infty^2/2.
\end{gathered} \tag{3.2}$$

We determined the density of the axial component of the force acting on a symmetrical wedge in a supersonic gas-suspension flow using the formula

$$(F - F_0)/2 = -p_n \sin\alpha + p_\tau \cos\alpha,$$

where F_0 is the density of the force with $\rho_p = 0$; p_n, p_τ are the components of the stress tensor. According to the law of conservation of momentum of the particles we have:

a) for elastic reflection

$$p_n = -(\delta_0\rho_0 u_0^2)\tilde{p} - 2\rho_p v_p^2, \quad p_\tau = 0;$$

b) with total absorption of the particle momentum by the surface

$$p_n = -(\delta_0\rho_0 u_0^2)\tilde{p}_0 - \rho_p v_p^2, \quad p_\tau = -\rho_p u_p v_p.$$

Figure 3 shows the results of the calculation of the force density in the case "a" (solid lines) and "b" (broken lines). For convenience, the dimensionless quantity $C_F = (F - F_0)/(2\delta_0\rho_0 u_0^2)$ is plotted along the ordinate axis.

The calculations were carried out with $Re_0 = 10$, $Pr = 0.65$, $c_p/c_s = 0.46$; $\alpha = 15°$, $M_\infty = 2.6$ for curve 1; $\alpha = 15°$, $M_\infty = 1.7$ for curve 2; and $\alpha = 30°$, $M_\infty = 2.6$ for curve 3. It is evident that as the distance from the front edge increases, the force density decreases down to the value determined from (3.2). The magnitude of the force in the case of an inelastic collision exceeds the value for an elastic collision, which is a result of the contribution of the tangential component of the momentum of the particles. Evidently, the presence of the admixture of particles in the supersonic flow can both increase and decrease the effective forces, depending on the linear size of the wedge L. Thus when $L \lesssim l_p(1)$ the drag increases with the gas-suspension flow over the wedge, while when $L \gg l_p(1)$ the drag decreases. This result is not unexpected, though it contradicts the intuitive idea of the origin of drag in the flow of particles. When $L \gg l_p(1)$ the two-phase flow is almost at equilibrium, and under these conditions the role of the particles reduces to changing the thermophysical parameters of the two-phase mixture. For example, $\tilde{\gamma} = (c_p + \delta_0 c_s)/(c_V + \delta_0 c_s) \approx \gamma[1 + \delta_0(1 - \gamma)c_s/c_p]$, i.e., the effective adiabatic index decreases in the presence of particles. The interaction

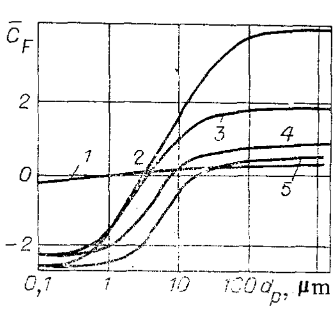

Fig. 4

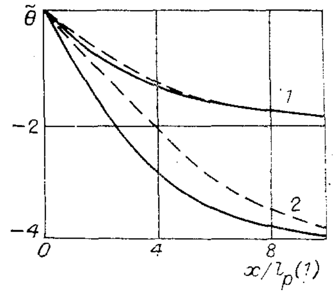

Fig. 5

of many particles causes the pressure behind the shock in the dusty gas to drop below that in the pure gas for fixed conditions of flow over the wedge (see [3, 4]).

The average drag in the flow of particles is $\bar{C}_F = \int_0^L C_F dx/L$. We calculated the dependences $\bar{C}_F(d_p)$ for different Mach numbers of the incident flow and different wedge angles, and for two mechanisms of momentum transfer (elastic and ineleastic impact). Some results of the calculations are presented in Fig. 4. Curves 1, 4, and 5 (elastic impact) correspond to $M_\infty = 2.6$, $\alpha = 15°$; $M_\infty = 2.6$, $\alpha = 30°$; $M_\infty = 1.7$, $\alpha = 15°$; curves 2 and 3 (inelastic impact) correspond to $M_\infty = 1.7$, $\alpha = 15°$; $M_\infty = 2.6$, $\alpha = 30°$. It is evident that there exist two sections of self-similarity relative to the size of the particles in the dependences $\bar{C}_F(d_p)$. Let us form the dimensionless parameter (Stokes number) $Sk = l_p(1)/L$. If $Sk \ll 1$, then we have an almost equilibrium flow and the drag decreases because of the pressure drop behind the shock [left self-similar section on the curve $\bar{C}_F(d_p)$]. On the other hand, if $Sk \gg 1$, then the drag is determined primarily by the momentum flux of the particles on the surface in the flow [the right self-similar section on the curve $\bar{C}_F(d_p)$]. An analogous behavior of the drag of a wedge in a subsonic gas-suspension flow was observed in the experiments in [11].

We studied the effect of reflected particles on the magnitude of the perturbation of the angle of inclination of the shock wave. Figure 5 shows the results of the calculations of $\tilde{\theta}$ as a function of the distance up to the tip of the wedge. The solid lines correspond to specular reflection and the broken lines correspond to sticking of the particles. The calculations were carried out for $Re_0 = 10$, $Pr = 0.65$, $c_p/c_s = 0.46$; for curves 1 $\alpha = 15°$, $M_\infty = 1.7$; and for curves 2 $\alpha = 30°$, $M_\infty = 2.6$. It is evident from Fig. 5 that at the tip of the wedge [at $x \lesssim l_p(1)$] the departure of the flow from equilibrium increases in the presence of reflected particles, but for small wedge angles the effect is insignificant. At a large distance from the tip of the wedge both flows being compared arrive at the same state; this follows directly from the asymptotic formulas (3.2).

LITERATURE CITED

1. G. G. Chernyi, Gas Flow at a High Supersonic Velocity [in Russian], Fizmatgiz, Moscow (1959).
2. V. V. Lunev, "Hypersonic flow of radiating gas over a wedge," Zh. Prikl. Mekh. Tekh. Fiz., No. 2 (1960).
3. R. A. Tkalenko, "Linear theory of supersonic flows of gas—particle mixtures," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 1 (1971).
4. G. A. Saltanov and R. A. Tkalenko, "Supersonic two-phase flow over a wedge," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 2 (1972).
5. G. A. Saltanov and A. V. Kurshakov, "Motion of particles behind an oblique shock in a supersonic two-phase flow over a wedge," Izv. Akad. Nauk SSSR, Energ. Transp., No. 1 (1971).
6. G. A. Saltanov, "Interaction of particles with the surface of a wedge in a flow," Izv. Akad. Nauk SSSR, Energ. Transp., No. 1 (1971).
7. A. P. Trunev and V. M. Fomin, "Two-phase flow of the gas—solid particles type over bodies taking into account erosion," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1983).
8. C. T. Crowe and D. T. Pratt, "Analysis of the flow field in cyclon separators," Comp. Fluids, 2, 249 (1974).
9. N. N. Yanenko, R. I. Soloukhin, A. N. Papyrin, and V. M. Fomin, Supersonic Two-Phase Flows under Conditions of Velocity Nonequilibrium of Particles [in Russian], Nauka, Novosibirsk (1980).
10. L. V. Ovsyannikov, Group Analysis of Differential Equations [in Russian], Nauka, Moscow (1979).

11. B. A. Balanin and V. A. Lashkov, "Drag of a flat wedge in a two-phase flow," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 2 (1982).

HYDRODYNAMIC INSTABILITY OF THE ABLATION FRONT IN THE PRESENCE OF ABLATION ACCELERATION OF A LAYER

N. A. Inogamov UDC 532.5+533.95

1. A large number of papers on the instability of an ablation front (AF) accompanying the acceleration of a layer by the ablation pressure has now been published [1-13]. The Cauchy problem, linearized around the stationary flow, which is found by numerical calculation, is studied numerically in [4, 5]; the numerical calculation of the Cauchy problem, linearized around the stationary flow, is carried out in [6, 7]. The stationary solution is found by numerical integration of a system of ordinary differential equations. We must make an important remark regarding [6-7]. We shall show that the stationary flow in a gravitational field has a peculiarity which invalidates the results of [6-7], regarding taking into account of the compressibility of the cold material and of the long-wavelength perturbations. We shall study the stationary solution in the region filled with cold matter. In this region, in the vicinity of the AF the flow is subsonic ($M \ll 1$). In the presence of gravity, the Mach number $M = v/c$ in the subsonic flow increases monotonically away from the AF in the cold matter and at some distance L_1 from the AF $M = 1$. The point is that in the cold matter the electronic thermal conductivity is small and the heat fluxes correspond as negligibly small. Therefore the stationary flow of cold matter is isentropic. For subsonic flow with $M \ll 1$ in the vicinity of the AF, because of the effect of the gravity, the pressure in the cold matter decreases away from the AF. The flow is isentropic, so that the density and the sound velocity decrease together with the pressure. The flow velocity v in this case increases, since the mass flow must be constant and correspondingly $M = v/c$ increases. The appearance of an internal supersonic zone in the stationary flow does not correspond to the essence of the problem of acceleration of the layer by the ablation pressure. For this reason the results of [6, 7] are useful only for $\lambda \ll L_1$. When $\lambda \simeq L_1$ the effect of compressibility of the cold matter becomes significant, but in the formulation of [6, 7] this effect is not taken into account correctly. The question of the compressibility and long-wavelength perturbations is analyzed in detail in this paper.

In addition to the works enumerated above, in which the linear stage is studied, interesting studies [7-9] on calculations of nonlinear two-dimensional flows have also been published; [1, 2, 10-12] concern analytical estimates. In [10, 11] it is proposed that a subsonic AF can be replaced by a jump in the deflagration wave. The work in [12] is based on a study of an unstable zone in which the vectors Δp and $\Delta \rho$ are antiparallel. It is assumed that the growing perturbations are spatially localized in this zone. We note that under the usual [3, 4] conditions ($I \simeq 10^{14}$ W/cm^2, Nd laser, layer thickness $L = 1$-4 μm) the thickness Δ_1 of this zone is small ($\Delta_1 \simeq 0.1$ μm). The growing perturbations can have $\lambda \gg \Delta_1$. In this case, the field of perturbations is spatially localized in a layer of thickness $\simeq\lambda \gg \Delta_1$ adjacent to the AF. In this case, the fine structure and the presence of the unstable zone are of no significance, since for such waves the fine structure is "hidden" within the thickness of the line (associated with the thickness of the "slate pencil") marking the perturbed boundary.

The short-wavelength scale of stabilization $\lambda_a = v_a^2/g \simeq M_a^2 L$, where $M_a = v_a/c_s$, v_a is the velocity of the AF relative to the cold matter, the index a indicates ablation, c_s is the sound velocity in the cold matter near AF, and g is the acceleration of the layer, is estimated in [1].

The effect of compressibility is analyzed in [2]. It is shown that for a typical large ratio of densities on the AF the dispersion curve in the case of isentropic gas coincides with the dispersion curve in the case of an incompressible liquid with an arbitrary ratio of the parameters $v_\lambda^2/c_s^2 \simeq \lambda/L$, where $v_\lambda = \sqrt{|g|\lambda}$.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 110-116, September-October, 1985. Original article submitted June 15, 1984.

 0021-8944/85/2605-0702$09.50